1. Introduction

The present work attempts to apply the fuzzy mathematical techniques in the field of economics, particularly in decision making problems. Most of the decision making problems are modeled using classical mathematical tools. However, there is a great amount of human reasoning that is intrinsically vague and uncertain, present in every decision making models. This demands a further exploration for better tools capable of expressing uncertainty and vagueness. In this work, several such decision making models are constructed using fuzzy and intuitionistic fuzzy mathematical tools.

2. Fuzzy Sets

A fuzzy set [Zadeh L., Fuzzy Sets, Information and Control, 8 (1965), pp.338-353], can be defined mathematically by assigning to each possible element in the universe of discourse, a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that element is similar or compatible with the concept represented by the fuzzy set. Thus, an element may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades very often lie in [0, 1].

3. Decision Making Models

The study of decision making, as the name suggests, is the study of how decisions are made, and how they can be made better or more successful. This also includes, in the broader ense, the choice or selection of alternatives from a given set of alternatives. Several important results have been published that justify the use of various criteria for decision making under uncertainty in the crisp environment. The most classical results are found in the works of Von Neumann and Morgenstern [Theory of Games and Economic Behavior, Princeton University Press, Princeton, NJ, 1947], and Savage ['The

Foundations of Statistics', Wiley, New York, (1954)] and the most commonly used criterion is the expected utility criterion axiomatized by Savage. In this utility maximization model, the subjective value attached to each consequence, as well as the degrees of confidence of the possible events commensurate can be quantified. Unfortunately, Savage's theory has been found to be inconsistent, in many ways, with how people actually behave. It is in this context Economist H. A. Simon ['Models of Man', Wiley, New York, 1955] enunciated his theory of 'bounded rationality' in 1955. He said that due to the cost or the practical impossibility of searching among all possible acts for the optimal, the decision maker simply looked for the first 'satisfactory' alternative that met some pre-specified target. His approach was called, the '*satisficing approach'*. E. Castognoli, M. Licalzi, R. Bordley etc. have unlade the assumption of a known target 't' by considering a random target 'T'. Recent works in the crisp and fuzzy case, on the target based models as well as their application and advantages, could be found in the works of A.E. Abbas, J.E.Mathewson, R.Bordley , C. Kirkwood, Van Nam Huynd, Yoshiteru Nakamon, E. Castagoli , M. Licalzi , etc.

Decisions can also be made using choice functions based on crisp and fuzzy preference relations. The rules for inducing exact choice sets from given fuzzy weak preference relations are called 'Fuzzy Preference Based Choice Functions' (FPBCF). S.A. Orlovsky, C.R. Barret, K. Basu, Z.Switalski, M.Roubens, S.Zahariev, etc., defined a number of such choice functions for ranking of alternatives. Rationality conditions of choice and rejection of the choice functions in the crisp case is given by Economist Amritya Sen and its counterpart in the fuzzy case by C.R.Barret.

Most of the criteria for decision making under uncertainty in the literature, share a common feature for ranking of alternatives, namely the commensurability of preference

scale and the uncertainty scale that are used in these criteria. But, works have also been done in decision making, which escapes the assumption of commensurability by C.Boutilier, D.Dubois, H.Fargier, P.Perny, H. R. Brafman, M.Tennenholtz, S.Tan, J. Pearl, Thomason etc. These decision models have wide practical applications in areas like Economics, Management studies etc. In these models, the alternatives can be pair-wise compared and ranked, but simultaneous ranking of alternatives is very tedious.

Another important area of decision making in economic problems is, decision making under uncertainty, in cases when inadequate information is available to the decision maker about the states and consequences. Many models have been proposed in literature in the crisp and fuzzy cases for preferences modeling in such cases. The most famous rule of this kind in the crisp case is the max-min rule of Wald, axiomatized by Arrow and Hurwicz, in the case when no information about the states is available to the decision maker. Some of the refinements of this criterion are by C.Boutilier, R.Brafman, D.Lehmann, M.Cohen , J.Y Jaffray, D.Dubois, H.Prade, R.Sabbadin,etc. A weighted extension of this criterion is the widely used Possibilistic Pessimistic Criterion (PPC), the optimistic counterpart of this criterion being the Possibilistic Optimistic Criterion (POC).These criteria are based on utility functions, which describe the preference of consequences for a decision maker, in economic problems. There have been many proposals for developing fuzzy theory of decision making by fuzzifying utility as given in the works of D.Dubois, H.Prade, K.Nakamura, S.A.Orlovosky, M.Roubens, etc.

In decision making problems, where non-membership values of the consequences are also available to the decision maker, alternatives can be ranked using Intuitionistic fuzzy sets. Intuitionistic fuzzy set introduced by Atannasov [Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*,20,(1986)87-96] is characterized by three functions, expressing

the degree of belongingness, degree of non-belongingness and the degree of hesitancy. Intuitionistic fuzzy set has been found to be highly useful to deal with uncertainty and vagueness and have become a popular topic of investigation in the fuzzy community. Works have been done on intuitionistic fuzzy decision making by H.Bustice, L.Lin, X.H.Yaun , Z.O.Xia , E.Szmidt, J.Kacprzyk, etc.

4. Chapter Summary

The thesis is organized into nine chapters. The opening chapter, **Chapter 0** is the **Introduction.** It gives an outline of the entire thesis.

All the basic concepts related to this work are discussed in **Chapter 1**.

Fuzzy decision making under uncertainty, in a satisficing environment, is analyzed in **Chapter-2**.Fuzzy models are constructed for ranking of alternatives in a satisficing environment, in both the cases i.e., when the target is fixed and when the target is uncertain to the decision maker, and results based on it are proved. Moreover, a model is constructed to update the fixed target in a decision problem, according to the changes that occur in the various factors of the decision problem during a given period of time. Examples are given as illustrations in all cases.

Chapter- 3 deals with decision making using fuzzy preference based choice functions. Here, the focus of discussion is on fuzzy preference based choice functions, whose score for the alternatives lie in [0,1]. Counter examples are given to illustrate that the existing choice functions fail to satisfy one or the other rationality conditions or behave improperly in some particular cases, even when the fuzzy preference relation is a fuzzy ordering. Thus, a new fuzzy preference based choice function, 'Dominance Against' Choice Function (DA-CF), is constructed. It is then proved that DA-CF satisfies all the

rationality conditions, when the admissible fuzzy preference relation is a fuzzy ordering. It is also very sensitive to even slight changes in the preference relation of the alternatives, which is a property very much required for ranking of alternatives in a fuzzy environment.

In **Chapter 4**, a model that is independent of the assumption of commensurability, namely the likely dominance rule, is proposed under the name 'Lifting Principle'. Using this rule, alternatives are pair-wise compared and ranked. But the major drawback is that, when the pair-wise ranked alternatives are to be ranked simultaneously, it is very tedious. Thus, in this work, models and algorithms are constructed both in the crisp and fuzzy cases for ranking the alternatives simultaneously in a very simple manner, in the case of alternatives that are pair-wise ranked using the possibility based dominance rule. In the fuzzy case, the algorithm is constructed using the choice function DA-CF with respect to the possibility based dominance function 'V', which is proved to be a fuzzy binary weak preference relation. Many fuzzy properties of 'V' have been proved. Also, it is proved that the choice function DA-CF with respect to 'V', satisfies all the rationality conditions. Numerical examples are given to illustrate the ranking of alternatives simultaneously using the constructed algorithms.

In **Chapter 5**, the set of alternatives D has been realized as a semigroup, by introducing a semigroup structure on D, using the fuzzy valuation function that has been defined in this chapter. New definitions for minimal elements, identity element, inverse, regular, commutative elements etc. are given in this context, and theorems related to it including a homomorphism theorem are proved.

In **Chapter - 6,** we consider decision making under uncertainty, in the case when the probability distribution about the states of nature is not known a priori but the

consequences are specifically quantified. Here decision making models are constructed in the fuzzy environment, based on the attitude of the decision maker whether optimist, pessimist, α-optimist or normal. The News Vendor problem which is an important problem in Business Economics has been solved in this context.

Chapter-7 deals with decision making under uncertainty, when inadequate information is available about the states and consequences. Here, we consider the Possibilistic Optimistic Criteria (POC) and the Possibilistic Pessimistic Criteria (PPC) which are commonly used in such cases. Even though, these criteria are the refinement of many criteria in the crisp case, they have many drawbacks as mentioned in this study. The fuzzy approach is very useful to handle such situations. Thus, new criteria which are the modifications of POC and PPC are constructed in the fuzzy environment, using the fuzzy aspiration, and fuzzy reservation degrees respectively, that are introduced in this chapter. Results on fuzzy aspiration and fuzzy reservation degree are proved. We have also proved that all the drawbacks in the existing criteria are rectified by the newly constructed criteria. Numerical examples are given to illustrate the same.

In **Chapter-8**, we explore the case when non membership values of the consequences are also available to the decision maker. Here intuitionistic fuzzy criteria for ranking of alternatives in an uncertainty environment are constructed, depending on the attitude of the decision maker whether optimist or pessimist. We show that when clear ranking of alternatives is not possible in the fuzzy and crisp cases, it is possible to clearly rank the alternatives in the intuitionistic fuzzy case. Examples are given to illustrate the same.

The last chapter, **Chapter- 9** is the **Conclusion**, which gives brief summary of the work.

5. Conclusion

In this study we have successfully modified a few existing models and also constructed new decision models in the fuzzy and intuitionistic fuzzy cases, for decision making in important economic problems: choice in a satisficing environment, choice based on preference relations, social choice problems, choice in the News Vendor problem, choice in problems where there is uncertainty in states and consequences, choice in problems where non membership values of the consequences are available to the decision maker etc. Also the set of alternatives have been realized as a semigroup, by using the fuzzy valuation function constructed and theorems including a homomorphism theorem have been proved in this context.

Signature of the Candidate **Signature of Research Guide**

Tutu M. John **Dr. Sunny Kuriakose A.**